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Current in Hubbard rings manipulated via magnetic flux

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Abstract

We study currents in a quantum ring threaded by a magnetic flux which is varied in an arbitrary way from an initial constant value ϕ_1 at time t_1 to a final constant value ϕ_2 at time t_2 . We analyze how the induced currents for $t > t_2$ can be controlled by the rate of flux variation $\dot{\phi} = (\phi_2 - \phi_1)/(t_2 - t_1)$. The dynamics of electrons in the ring is described using the Hubbard and the extended Hubbard models. In the Hubbard model with infinite on-site repulsion the current for $t > t_2$ is shown to be independent of the flux variation before t_2 and is fully determined by a solution of the initial equilibrium problem and by the value ϕ_2 of the flux. For intermediate values of the interaction strength the current displays regular or irregular time oscillations and the amplitude of oscillations is sensitive to the rate of the flux changing $\dot{\phi}$: slow changes of the flux result in small amplitudes of the current oscillations and vice versa. We demonstrate that the time dependence of the induced current bears information on electronic correlations. Our results have important implications for not only mesoscopic rings but also the designing of quantum motors built out of ring-shaped optical lattices.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Time-dependent manipulation of quantum states in nanosystems is an important problem directly related to future applications both in the context of quantum control [1] and in the reduction of decoherence [2]. There is a natural area for implementing such ideas: mesoscopics and nanoscopics where quantum effects play a crucial role [3]. Unfortunately, analysis of quantum systems affected by external time-dependent forces and/or fields $F(t)$ is extremely difficult and only very few models are exactly solvable [4]. The best known examples concern a driven quantum oscillator [5–7] and a two-level system in a circularly polarized magnetic field [8, 9]. In many cases the solvability is related to certain symmetries of the system. Moreover, there are interesting regimes of a strong external driving $F(t)$ when the linear response theory cannot be applied and nonlinear responses as well as time-dependent phenomena govern the system dynamics. For a time-periodic driving $F(t)$ one may apply the Floquet theory [10] which in many cases provides only approximate numerical results. However, there is a Floquet-theory-based approach which admits an exact analytical solution at arbitrary

external driving [11]. Another problem concerns the possibility of experimental verification of theoretical predictions. Here a pertinent question arises: how does the final (generally non-equilibrium) state depend on the particular shape of the driving $F(t)$? For the purpose of practical applications it is desirable to find systems which are robust against small temporal changes of $F(t)$, e.g., originating from imperfect realization of the assumed conditions. A realistic example will be pointed out in this work.

We study currents in a one-dimensional (1D) quantum system of ring topology threaded by a time-dependent magnetic flux $\phi = \phi(t)$ generated by a magnetic field $B = B(t)$ (see figure 1). If the magnetic field is perpendicular to the ring, a variety of Aharonov–Bohm effects can be observed. As an example one can mention persistent currents which, in the case of a static magnetic field B , oscillate as a function of the flux ϕ [12]. When the magnetic flux changes linearly with time, $\phi(t) = At$, the induced electromotive force $E \propto d\phi(t)/dt = A$ is time independent and electrons move in a static electric field. This case is interesting because it is similar to a system of electrons moving in a periodic lattice under the influence of a dc voltage. In the regime of small values of

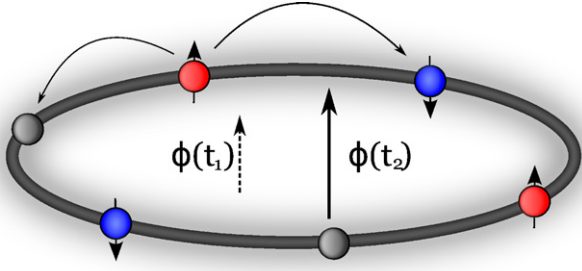


Figure 1. The scheme of the example system studied: the quantum ring consists of $N = 6$ sites and four electrons, two with spin down and two with spin up, $n_{\uparrow} = 2, n_{\downarrow} = 2$. The ring is threaded by the magnetic flux ϕ which is varied in an arbitrary way from the initial constant value $\phi(t_1)$ at time t_1 to the final constant value $\phi(t_2)$ at time t_2 . Electrons can jump to nearest neighboring sites and two electrons with opposite spins can occupy the same site. The induced currents are studied for times $t > t_2$ and in some regimes can be controlled by the rate of flux variation $\dot{\phi} = [\phi(t_2) - \phi(t_1)]/(t_2 - t_1)$.

At the induced currents display Bloch oscillations [13] and the time-averaged current is zero. It is true at zero temperature and in the dissipationless regime (the case of elastic scattering of electrons). In the case of inelastic scattering of electrons, a dc current can occur [14]. Also for large values of A the induced currents may display a dc component [15, 16]. The last example that we want to mention is a current driven by a periodic magnetic flux $\phi(t) = \phi_{dc} + \phi_{ac} \cos(\omega t)$ [17]. In this case, a dc persistent current is of the same order as in the static magnetic field and the oscillating current is very small, containing in its Fourier spectrum a significant contribution from only a small number of frequencies [18].

Here, we propose another scheme involving variation of the magnetic flux to manipulate currents (see figure 1): the magnetic field perpendicular to the ring is changed from its initial constant value $B(t_1)$ to the final constant value $B(t_2)$. The corresponding magnetic flux is changed from the initial constant value $\phi(t) = \phi(t_1) = \phi_1$ for time $t \leq t_1$ to the final constant value $\phi(t) = \phi(t_2) = \phi_2$ for $t \geq t_2$. The dynamics of electrons moving in the ring is described by the Hubbard and the extended Hubbard models. We analyze how the induced currents depend on the rate of flux variation $\dot{\phi} = (\phi_2 - \phi_1)/(t_2 - t_1)$. We find that for zero and infinitely strong many-body interactions the resulting current has the following properties: for $t > t_2$ it is independent of $\phi(t)$ for $t_1 < t < t_2$, i.e., the resulting current is independent of the way in which the magnetic flux is modified/switched on; the solution of the equilibrium problem at $t = t_1$ and the value of ϕ_2 entirely determine the current for $t \geq t_2$. For intermediate values of the interaction strength we carry out numerical calculations and point out that the current displays regular or irregular time oscillations and the amplitude of the oscillations is sensitive to the rate of the flux changing $\dot{\phi}$: slow changes of the flux result in small amplitudes of the current oscillations and vice versa. We conclude that in some regimes the currents can be controlled by the rate of flux variation $\dot{\phi}$.

The 1D Hubbard-type Hamiltonians with time-dependent flux have been investigated numerically in the context of various physical phenomena. The analyzed time dependence

of the flux was directly connected with the problems investigated. In particular, $\phi(t) \propto t$ has been discussed in [16] as a driving mechanism for the metal–insulator transition, whereas $\phi(t) \propto \exp(-at^2) \cos(\omega t)$ has been used in a recent study of light-pulse excitations in one-dimensional Mott insulators [19]. It has also been argued for a system of non-interacting carriers that currents can be generated by applying two linearly polarized unipolar pulses [20, 21].

Our analysis can be applied either to mesoscopic rings or to rings built in the optical lattice setup [22]. The difference in energy scales of the two systems shows up mainly in different time scales of the external driving.

The plan of the paper is as follows. In section 2 we consider non-interacting particles moving in a nanoring threaded by a varying magnetic flux. Section 3 contains analysis of currents in Hubbard rings. In section 4 we analyze currents in the limiting regime of the infinite Coulomb repulsion. Transport of particles within the extended Hubbard model is studied in section 5. Finally, section 6 provides a summary and some conclusions.

2. Non-interacting particles

Although the method of reasoning that we apply for non-interacting fermions is rather trivial, it nicely illustrates the general method which is applicable also to a non-trivial case of correlated electron systems. Therefore, we start with the Hamiltonian of non-interacting particles in the ring threaded by the magnetic flux. It is a sum of one-particle Hamiltonians

$$H(t) = \frac{1}{2m} \left[p - \frac{e}{L} \phi(t) \right]^2, \quad (1)$$

where L is the circumference of the ring and e is the charge of the particle. The current operator is related to the momentum observable in the following way:

$$I(t) = -\frac{\partial H(t)}{\partial \phi(t)} = \frac{e}{mL} \left[p - \frac{e}{L} \phi(t) \right]. \quad (2)$$

Now, let us assume that the magnetic flux ϕ is varied in an arbitrary way from an initial value ϕ_1 at time t_1 to a final value ϕ_2 at time t_2 . One can explicitly extract the time-dependent part of the current operator

$$I(t) = I_1 + \Delta I(t), \quad (3)$$

where

$$I_1 = \frac{e}{mL} \left[p - \frac{e}{L} \phi(t_1) \right], \quad (4)$$

$$\Delta I(t) = \frac{e^2}{mL^2} [\phi(t_1) - \phi(t)]. \quad (5)$$

The averaged current flowing in the ring is determined by the relation

$$\langle I(t) \rangle = \text{Tr}(\rho(t)I(t)), \quad (6)$$

where $\rho(t)$ is a density matrix of the system. Its time evolution is determined by the von Neumann equation

$$i\hbar \dot{\rho}(t) = [H(t), \rho(t)]. \quad (7)$$

Note that from equation (1) it follows that

$$[H(t_a), H(t_b)] = 0 \quad (8)$$

for arbitrary times t_a and t_b . In consequence, a solution of the von Neumann equation has the form

$$\rho(t) = \exp\left[-\frac{i}{\hbar} \int_{t_0}^t H(s) ds\right] \rho(t_0) \exp\left[\frac{i}{\hbar} \int_{t_0}^t H(s) ds\right]. \quad (9)$$

If $\rho(t_0)$ commutes with the Hamiltonian $H(t)$ for any $t \geq t_0$ then $\rho(t) = \rho(t_0)$. The latter requirement is not very restrictive. It is fulfilled by the Gibbs state

$$\rho(t_0) = \frac{e^{-\beta H(t_0)}}{\text{Tr}[e^{-\beta H(t_0)}]} \quad (10)$$

and, from the experimental point of view, seems to be the easiest and the most natural choice of the initial preparation. Let us take $t_0 = t_1$. Then the averaged current reads

$$\langle I(t) \rangle = \text{Tr}(\rho(t)I(t)) = \text{Tr}[\rho(t_1)I_1] + \Delta I(t), \quad (11)$$

where the first term on the rhs is the initial equilibrium persistent current and the second term is the current induced by a time-dependent component $\Delta I(t)$. As the latter quantity is independent of the initial state, one can easily calculate the current induced in a system of n non-interacting particles $\Delta I_n(t) = n\Delta I(t)$. It is instructive to compare $\Delta I_n(t)$ with the maximal amplitude of persistent currents I_{PC} at zero temperature [12]. One finds

$$\frac{\langle \Delta I_n(t) \rangle}{I_{PC}} = 2 \frac{\phi(t_1) - \phi(t)}{\phi_0}, \quad (12)$$

where $I_{PC} = v_F e/L$, $v_F = \hbar\pi n/(mL)$ is the Fermi velocity and $\phi_0 = h/e$ is the flux quantum.

Let us notice remarkable properties resulting from equation (12), namely:

- (1) the averaged current $\langle I(t) \rangle$ depends on the flux only at the same instant of time t , i.e., the current is independent of the way in which the magnetic flux is switched on;
- (2) $\langle I(t) \rangle$ is fully determined by the solution of the initial equilibrium problem;
- (3) one can induce currents which are significantly larger in amplitude than the maximal persistent currents provided that $\phi(t) - \phi(t_1) \gg \phi_0$.

3. Hubbard rings

It is known that electrons in 1D systems are almost always strongly correlated [23]. In that sense, the free electron approximation applied to 1D quantum rings is disputable because strong electronic correlations may substantially modify the properties of persistent currents; see e.g. review [24]. In the following we discuss to what extent the conclusions derived for free fermions ((1)–(3)) are applicable

to more realistic systems of correlated particles. First, we consider the 1D Hubbard model [23, 25]

$$H_H(t) = -J \sum_{j,\sigma} (e^{i\tilde{\phi}(t)} a_{j+1,\sigma}^\dagger a_{j,\sigma} + \text{h.c.}) + U \sum_j n_{j,\uparrow} n_{j,\downarrow}, \quad (13)$$

where J is the hopping integral, U is the on-site Coulomb repulsion, $n_{j,\sigma} = a_{j,\sigma}^\dagger a_{j,\sigma}$ and $\sigma = \uparrow, \downarrow$. For the ring consisting of N sites the dimensionless magnetic flux $\tilde{\phi}(t) = 2\pi\phi(t)/N\phi_0$. The current operator reads

$$I(t) = i \frac{2\pi J}{N\phi_0} \sum_{j,\sigma} (e^{i\tilde{\phi}(t)} a_{j+1,\sigma}^\dagger a_{j,\sigma} - \text{h.c.}). \quad (14)$$

We choose J as the energy unit, whereas time and current will be expressed in units of $\tau = \hbar/J$ and $I_0 = 2\pi J/N\phi_0$, respectively.

Before we carry out discussion based on analytical results, it is instructive to inspect numerical studies. We have considered a current induced by a linear (in time) change of magnetic flux from the initial constant value $\phi(t_1 = 0) = 0$ to the final constant value $\phi(t_2) = \phi_0$ (see figure 2). Under equilibrium conditions the partition function is a periodic function of flux with the period ϕ_0 . Therefore, the equilibrium persistent currents for $\phi = 0$ and ϕ_0 are equal. This was the motivation for our choice of $\phi(t_2)$.

The time dependence of the average current $\langle I(t) \rangle$ has been calculated from equation (6) via a solution of the von Neumann equation for $\rho(t)$ under an initial condition being the equilibrium state $\rho(t_1) \propto \exp(-\beta H_H(t_1))$ with $\beta \rightarrow \infty$. Results presented in figure 2 have been obtained for a ring consisting of $N = 6$ sites with various numbers of spin up n_\uparrow and spin down n_\downarrow particles. The general results can be formulated as follows: in the small and large U/J limits, a dc current is observed for $t > t_2$. Its amplitude is independent of the rate of flux variation $\dot{\phi} = (\phi_2 - \phi_1)/(t_2 - t_1)$ and is greater than the maximal value of the equilibrium persistent current. For moderate values of U/J , the current displays regular or irregular time oscillations. However, its average over time is non-zero and the dc component can be detected. The typical frequency of the oscillations depends on the electron correlations: for stronger correlations, i.e., for larger U/J , the frequency of the current is higher. In contrast, the amplitude decreases as U/J increases. The amplitude of the oscillations is more sensitive to the rate of the flux changing $\dot{\phi}$: slow changes of the flux result in small amplitudes of the current oscillations and vice versa.

From figure 2 we conclude that more regular behavior of the current versus time can be observed for a symmetric case $n_\uparrow = n_\downarrow$ (the left panel of figure 2). The findings are as follows:

- (i) Two characteristic regimes can occur: one with a strict dc component of the current (a negligibly small part of the alternating current) $\langle I(t) \rangle = I_{dc}$ and one with the regular time-oscillating component, $\langle I(t) \rangle = I_{dc} + I_{ac}(t)$.
- (ii) The dc current occurs in two limiting regimes of very weak and very strong electron–electron repulsion (the cases $U = 0$ and $16J$).

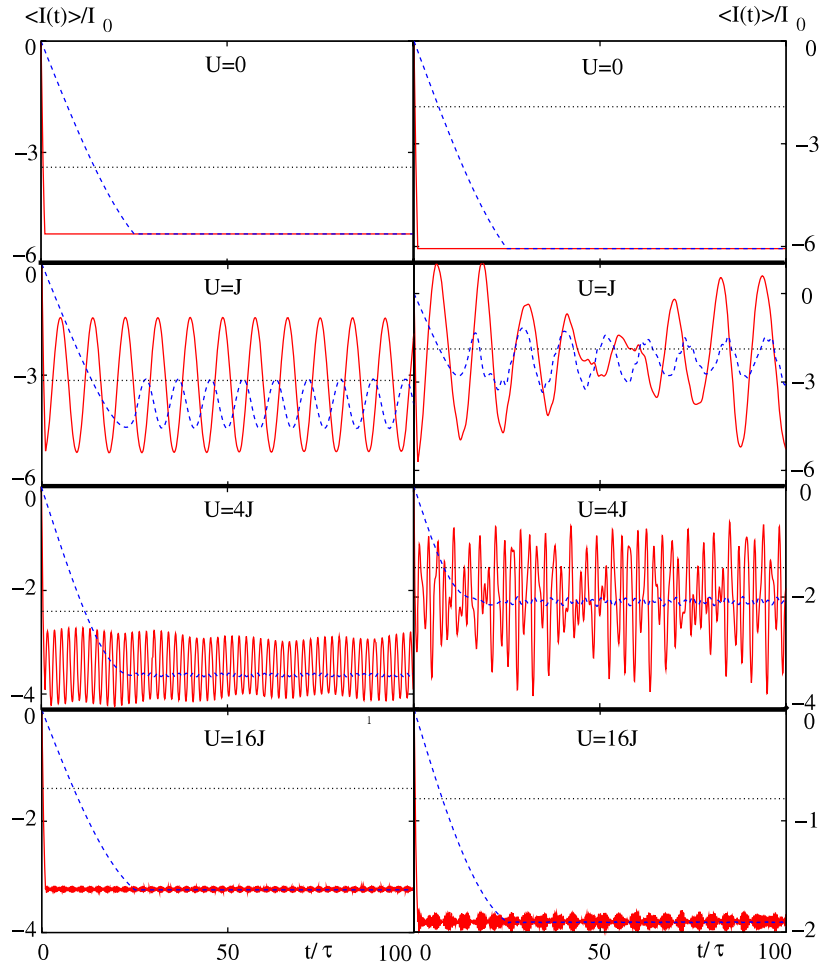


Figure 2. The dimensionless averaged current $\langle I(t) \rangle / I_0$ induced by the change of the magnetic flux $\phi(t)$ from the initial constant value $\phi(t_1 = 0) = 0$ to the final constant value $\phi(t_2) = \phi_0$ with the rate $\dot{\phi}(t) = \text{const}$ for $t \in (t_1, t_2)$. The results are obtained for the ring consisting of $N = 6$ sites and four electrons, $n_\uparrow = 2, n_\downarrow = 2$ (left panels); and five electrons, $n_\uparrow = 3, n_\downarrow = 2$ (right panels). Continuous (red) lines show results for $t_2 = \tau$ and dashed (blue) lines for $t_2 = 25\tau$. The horizontal dotted lines show the maximal values of the equilibrium persistent current.

(iii) The regular ac current with a non-zero dc component is detected for the case $U = J$, i.e. when two energy scales in the system (13) are the same.

In section 5 we discuss how the general characteristics of the induced currents depend on the number of charge carriers. Another pertinent question concerns the scaling of the induced current with the size of the system. The latter problem goes beyond the scope of the present work.

4. Currents in the infinite- U Hubbard model

The numerical results suggest that the conclusions (1) and (2) formulated at the end of section 2 for free particles hold true also for the system described using the Hubbard Hamiltonian (13) with $U = 0$ or $U/J = \infty$. The former case ($U = 0$) is again trivial, since in the Bloch representation one gets

$$H_H(t) = - \sum_{k,\sigma} 2J \cos(k - \tilde{\phi}(t)) a_{k,\sigma}^\dagger a_{k,\sigma}. \quad (15)$$

As equation (8) holds true for the above Hamiltonian, $\rho(t) = \rho(t_1) = \text{const}$ and $\langle I(t) \rangle$ is independent of the magnetic flux

$\phi(t')$ for $t' < t$. Below we present simple calculations, which explicitly show that currents induced in the $U \rightarrow \infty$ Hubbard model have the same properties as the currents in a system of non-interacting fermions. In the case of infinitely strong interaction one can rewrite the Hamiltonian (13) in the form

$$H_H(t) = H_L(t) + H_R(t), \quad (16)$$

$$H_L(t) = H_R^\dagger(t) = -J e^{i\tilde{\phi}(t)} P \sum_{j,\sigma} a_{j+1,\sigma}^\dagger a_{j,\sigma} P, \quad (17)$$

where the operator $P = \prod_{j=1}^N (1 - n_{j,\uparrow} n_{j,\downarrow})$ projects out states with doubly occupied sites. It is clear that $[H_L(t_1), H_L(t_2)] = [H_R(t_1), H_R(t_2)] = 0$. Then, in order to prove that equation (8) holds true, it is enough to show that $[H_L(t_1), H_R(t_2)] = 0$. One finds

$$[H_L(t_1), H_R(t_2)] = J^2 e^{i(\tilde{\phi}(t_1) - \tilde{\phi}(t_2))} (A - B), \quad (18)$$

$$A = \sum_{i,j,\sigma,\mu} P a_{i+1,\sigma}^\dagger a_{i,\sigma} \bar{P} a_{j,\mu}^\dagger a_{j+1,\mu} P, \quad (19)$$

$$B = \sum_{i,j,\sigma,\mu} P a_{j,\mu}^\dagger a_{j+1,\mu} \bar{P} a_{i+1,\sigma}^\dagger a_{i,\sigma} P. \quad (20)$$

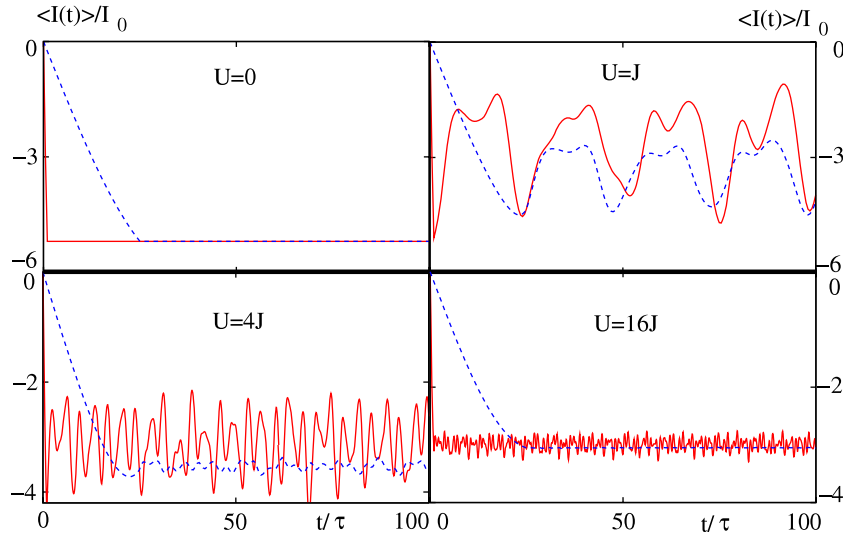


Figure 3. The same as in the left column of figure 2 but for the extended Hubbard model with $V = 0.4U$.

We introduce the notation $\bar{P} = P$ because it is convenient to distinguish different positions of these operators. Analyzing the operator A one can note that because of the presence of the projection operators P one can neglect \bar{P} unless the hopping $a_{i+1,\sigma}^\dagger a_{i,\sigma}$ removes double occupancy generated by $a_{j,\mu}^\dagger a_{j+1,\mu}$. A similar method of reasoning applies to the operator B . Therefore, in equations (19) and (20) one can replace \bar{P} with $(1 - \delta_{ij}) + \delta_{ij} \bar{P}$. It is also important to note that $a_{i,-\sigma} \bar{P} a_{i,\sigma}^\dagger |\psi\rangle = 0$ for arbitrary state $|\psi\rangle$. Taking into account these properties one gets

$$\begin{aligned} A - B &= \sum_{i,j,\sigma,\mu} (1 - \delta_{ij}) P [a_{i+1,\sigma}^\dagger a_{i,\sigma}, a_{j,\mu}^\dagger a_{j+1,\mu}] P \\ &+ \sum_{i\sigma} P a_{i+1,\sigma}^\dagger a_{i,\sigma} \bar{P} a_{i,\sigma}^\dagger a_{i+1,\sigma} P \\ &- \sum_{i\sigma} P a_{i,\sigma}^\dagger a_{i+1,\sigma} \bar{P} a_{i+1,\sigma}^\dagger a_{i,\sigma} P. \end{aligned} \quad (21)$$

The first term vanishes because the commutator is proportional to δ_{ij} . Now, the projection operators \bar{P} in the second and third terms can be replaced by $1 - n_{i,-\sigma}$ and $1 - n_{i+1,-\sigma}$, respectively. Then, one gets

$$\begin{aligned} A - B &= P \sum_{i\sigma} n_{i+1,\sigma} (1 - n_{i,\sigma}) (1 - n_{i,-\sigma}) P \\ &- P \sum_{i\sigma} n_{i,\sigma} (1 - n_{i+1,\sigma}) (1 - n_{i+1,-\sigma}) P. \end{aligned} \quad (22)$$

One can see that $A - B$ is expressed as a difference of two operators (the first and second lines in the above equation). The first one counts how many occupied sites *succeed* empty sites, whereas the latter counts how many occupied sites *precede* empty sites. In a ring-shaped system these numbers are equal for an arbitrary state. Hence $A - B = 0$, equation (8) holds true and, consequently, changing the flux does not influence the density matrix.

It is instructive to discuss our results in the framework of the standard approach to the 1D Hubbard model via the Bethe ansatz [26]. The latter provides a solution for all interaction strengths and band-fillings and we refer the reader to [27] for a

comprehensive review on the equilibrium properties of the 1D Hubbard model. In particular, the elementary excitations are expressed in terms of holons and spinons which, in general, interact and are not independent [27]. Then, analysis of the response functions within the Bethe ansatz is far from straightforward. The complete charge–spin separation over all energy scales occurs only in the case $U \rightarrow \infty$ [28]. As the vector potential couples to charged orbital degrees of freedom, in the case of the full spin–charge separations the system responds to time-dependent flux in the very same way as a system of non-interacting fermions.

5. Currents in the extended Hubbard model

In this section, we study currents in a more general system which in contrast to the previous one is non-integrable [29]. For this purpose we have carried out numerical calculations for the extended Hubbard model with the Hamiltonian³

$$H_{EH} = H_H + V \sum_{j\sigma\mu} n_{j,\sigma} n_{j+1,\mu}, \quad (23)$$

where V is the potential of the nearest neighbor Coulomb repulsion.

Figure 3 shows results similar to those presented in the left column of figure 2 but calculated for the extended Hubbard model with $V = 0.4U$. A comparison of figures 2 and 3 suggests that the general properties of $\langle I(t) \rangle$ derived for the non-interacting system hold true also for the extended Hubbard model, at least in some regimes of the model parameters. As can be inferred from figure 3, fluctuations of $\langle I(t) \rangle$ for $t > t_2$ gradually extinguish when the interactions become stronger and the magnitude of these oscillations decreases when t_2 increases. Simultaneously, for $U \gg J$ the magnitude of the

³ For $U/J \rightarrow \infty$, the extended Hubbard model can be mapped onto the model of spinless fermions with nearest neighbor repulsion which, in turn, can be mapped onto the XXZ Heisenberg model and then analyzed via the Bethe ansatz. For details, see e.g., [30].

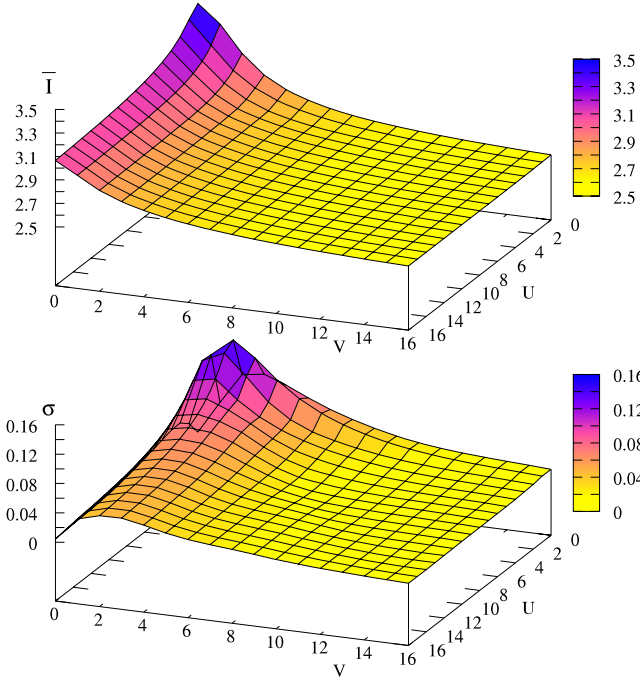


Figure 4. \bar{I} (upper panel) and σ (lower panel) as a function of U and V for the extended Hubbard model (see equation (24)). The current is induced by the change of magnetic flux $\phi(t)$ from $\phi(t_1 = 0) = 0$ to $\phi(t_2) = \phi_0$ with $\dot{\phi}(t) = \text{const}$ for $t \in (t_1, t_2)$. The results are obtained for $N = 6$, $n_\uparrow = 1$, $n_\downarrow = 1$ and $t_2 = \tau$.

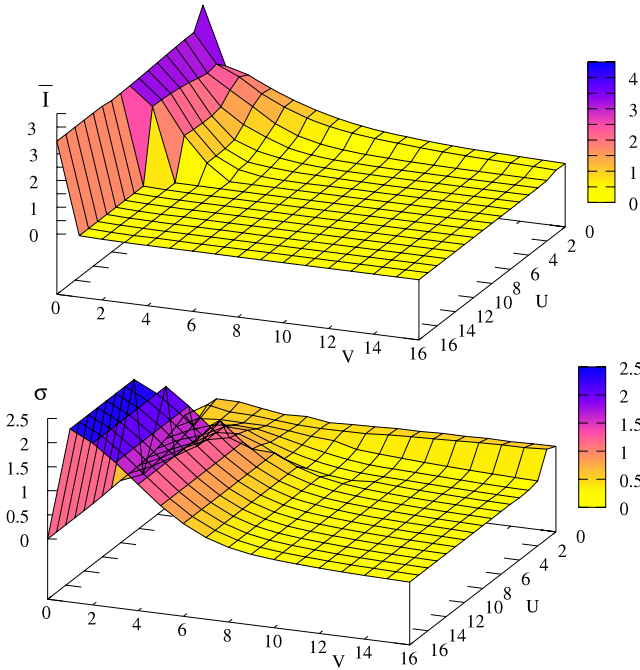


Figure 5. The same as in figure 4 but for $n_\uparrow = 2$, $n_\downarrow = 1$.

current for $t > t_2$ becomes almost independent of t_2 . Most of the previous studies on the 1D extended Hubbard model considered the half-filled [31–39] and/or quarter-filled [40–45] cases. In the former there is a transition from a Mott insulator to charge density wave insulator at $U \simeq 2V$, whereas in the latter case a metal–insulator transition takes place at finite V .

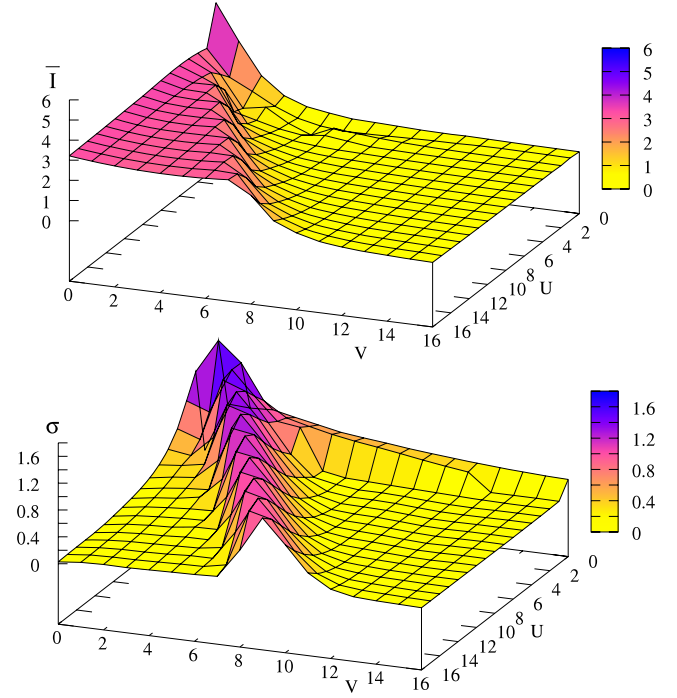


Figure 6. The same as in figure 4 but for $n_\uparrow = 2$, $n_\downarrow = 2$.

The phase diagrams obtained are complex and include charge density waves, spin density waves, superconductivity and regions with phase separation. Therefore, one may expect the currents induced by change of magnetic flux to be sensitive to various model parameters. In order to investigate this problem it is necessary to describe the induced current in a quantitative way. As demonstrated above, the general properties of the current can be characterized by the dc component and the oscillatory part, $\langle I(t) \rangle = I_{\text{dc}} + I_{\text{ac}}(t)$. Therefore, we have calculated its time-averaged value \bar{I} and the standard deviation σ defined as

$$\bar{I} = \frac{1}{t_m - t_2} \int_{t_2}^{t_m} dt \langle I(t) \rangle, \quad \sigma^2 = \overline{\langle I(t) \rangle^2} - \bar{I}^2. \quad (24)$$

In the analysis presented we have chosen $t_m = 100\tau$. The time-averaged current corresponds to the dc component and the standard deviation corresponds to the amplitude of the ac current. The quantum-mechanical expectation value $\langle I(t) \rangle$ for the current operator is defined in equation (6).

Figures 4–7 show the characteristics of the induced currents for $n_\uparrow + n_\downarrow = 2, 3, 4$, and 5, respectively. Below quarter-filling (figure 4), interactions affect the current in a similar way to in the regular Hubbard model. The strongest current fluctuations occur for the case where the interactions strengths are of the order of the hopping integral. For large values of U and/or V the time-averaged current remains significant, whereas σ becomes negligibly small. The situation at quarter-filling is very different, as can be inferred from figure 5. For large U , the nearest neighbor repulsion tends to localize the particles. Here, the current fluctuation can exceed its time-averaged value. The most interesting results occur for the electron concentration between quarter-filling and half-filling (see figures 6 and 7). For this doping, the significant dc

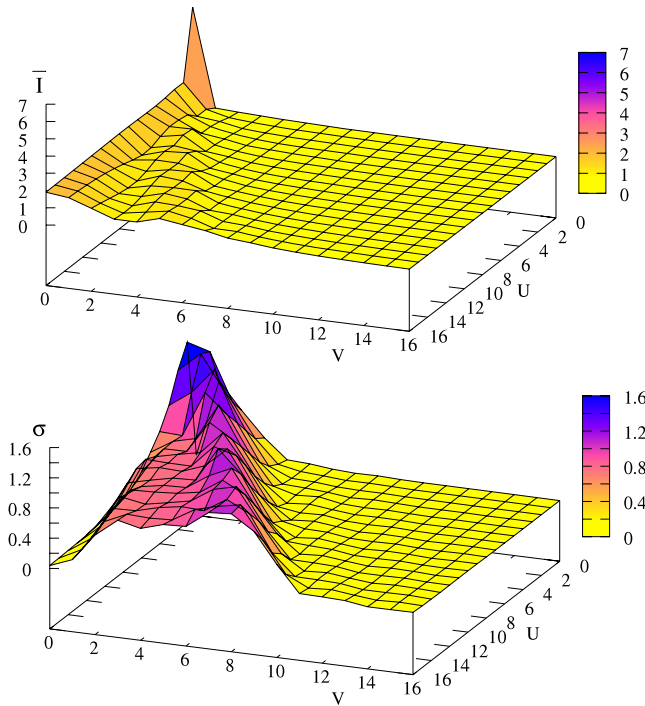


Figure 7. The same as in figure 4 but for $n_{\uparrow} = 3, n_{\downarrow} = 2$.

currents can be induced only for $V \ll U/2$, whereas the largest current fluctuations occur for $V \simeq U/2$. Analyzing results in figures 4–7, one can observe a common feature: the current fluctuations are strongest in the regimes where the gradient $\nabla \bar{I}(U, V)$ is large.

6. Conclusions

We have analyzed currents induced by temporal changes of the magnetic flux piercing a quantum ring. On the one hand, experimental observations of the current may give important insight into various properties of the system, e.g. the spin-charge separation. On the other hand, one can induce an oscillating current of desired amplitude and frequency, whose time average is non-zero and contains a dc component. The significant advantage of the method based on the flux variation is the ‘non-invasive’ manipulation performed outside the ring, without coupling to external leads. Recent progress in the highly controlled fabrication of quantum ring structures makes the verification of our findings quite realistic for the near future. With respect to this point it is also necessary to recall the significant progress in laser physics that enables tuning of the femtosecond electromagnetic pulses. As discussed in [46], the femtosecond pulse shaping allows for generating nearly arbitrarily shaped ultrafast optical waveforms.

We should note that basic limitations (time shorter than the relaxation time and the flux of the order of the flux quantum) are not so restrictive for experiments performed in the optical lattice setup. In this context, our results pose some important implications for the design of quantum motors discussed in [22]. In the Hubbard model with $U/J \ll 1$ or $U/J \gg 1$ the current instantaneously follows the flux. This holds true

also in some specific regimes of the extended Hubbard model, which have been studied in the preceding section. In these regimes, significant dc currents can be generated neither by impulses with $\phi(t_2) \simeq \phi(t_1)$ nor by a magnetic flux that has small time-averaged value $\overline{\phi(t)} \ll \phi_0$. In the former case the final current is the same as the initial one, whereas in the latter case the time-averaged value of the current (dc component) should be small. Consequently, in order to obtain the best performance of the ac-driven quantum motors one should focus on systems where the interaction strengths are of the order of the hopping integral. However, generation of significant dc currents in a system with $U/J \ll 1$ or $U/J \gg 1$ is possible provided $\phi(t_2) - \phi(t_1) \sim \phi_0$. In this case, the induced current is independent of the way in which the magnetic flux is modified/switched on, i.e., the current is rather insensitive to imperfect realization of the assumed time dependence of the flux.

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